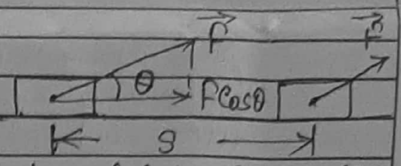
* Work, Energy and Power * "1. B.K.Sto"

Work :- Work is said to be done by or against a force when the point of application of a body is displaced in or opposite to the applied force.

Work done is measured by the dot product of force and displacement.

i.e. $W = \vec{F}\cdot\vec{s}$

or, $\boxed{W = FS\cos\theta}$



Hence work done may also be defined as the effective component of force and displacement

or

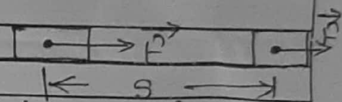
Work done is the product of component of force in the direction of displacement and magnitude of displacement.

Sp. Cases :-

(i) If $\theta = 0°$ i.e. the force acts along the direction of motion, then

$W = FS\cos 0°$

$\therefore \boxed{W = FS}$

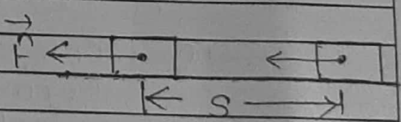


Thus, work is said to be done by a force when a force acts on a body and the body moves through some distance in the direction of the force.

(ii) If $\theta = 180°$ i.e. the force acts opposite to the direction of motion, then

$W = FS\cos 180°$

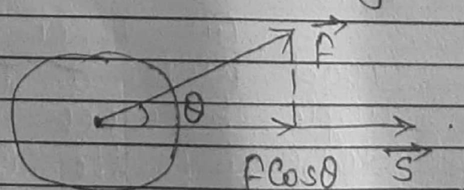
$\therefore \boxed{W = -FS}$



Thus work is said to be done against a force when force acting in the opposite direction of displacement.

Positive work :- If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive.

For positive work,
θ is acute i.e. θ is less than $90°$.



$\therefore W = Fs\cos\theta$ = a positive value.

Examples:-

(i) When a body falls freely under the gravity ($\theta = 0°$), the work done by gravity is positive.

(ii) When horse pulls a Cart, the work done by horse is positive.

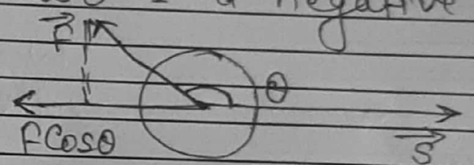
(iii) When a spring is stretched, both the stretching force and the displacement act in the same direction. So work done is positive.

etc.

Negative work:- If a force acting on a body has a component in the opposite direction of displacement, the work done is negative.

Now, for negative work, θ is obtuse i.e, θ is greater than $90°$.

i.e, $W = Fs\cos\theta$ = a negative value



Examples:

(i) When a body is lifted, the work done by gravitational force is negative.

(ii) When a body slides against a rough horizontal surface, its displacement is opposite to the force of friction. The work done by friction is negative.

etc.

Zero work:- Work done by force is zero if the body gets displaced along a direction perpendicular to the direction of applied force. Also, the work done is zero if $\vec{F}$ or $\vec{s}$ or both are zero.

Examples:- (i) For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other. So the work done by centripetal force is zero.

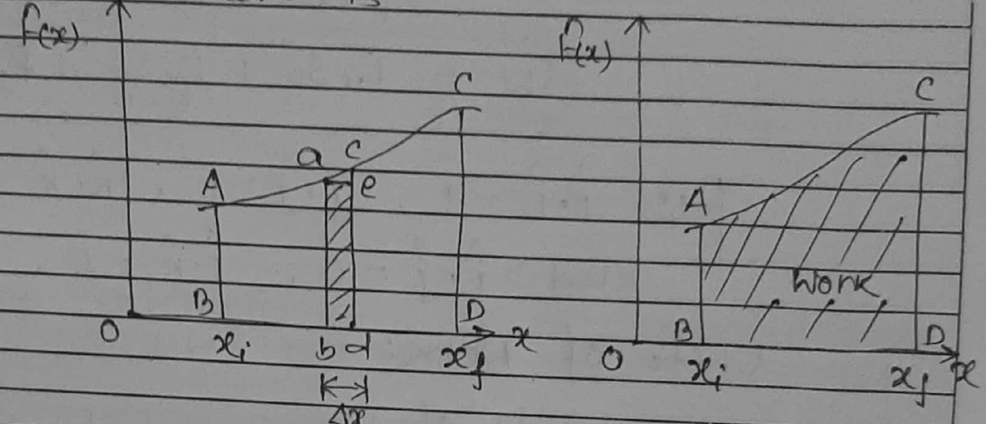
(ii) When a coolie walks on a horizontal road with a load on his head, then the angle between $\vec{F}$ and $\vec{s}$ is $90°$. So work done by the coolie on the load is zero.

(iii) The work done in pushing an immovable stone is zero, because the displacement of stone is zero.

Work done by a variable force.

Consider a variable force F acts on a body along the fixed direction, say x-axis. The magnitude of the force F depends on x, as shown by force-displacement graph. Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under the force F.

The displacement can be divided into a large number of small equal displacements Δx. During small displacement Δx, the force F can be assumed to be constant. Then the work done is



$$W = F\Delta x = \text{Area of rectangle abde}$$

Therefore, the total work done is

$$W = \sum_{x_i}^{x_f} F\Delta x$$

= Sum of the areas of all rectangles erected over all the small displacements.

= Area of ABDC

In the limit when $\Delta x \to 0$, the number of rectangles tends to be infinite, but above summation approaches a definite integral whose value is equal to the area under the curve. Thus, the total work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F\Delta x = \int_{x_i}^{x_f} F\,dx$$

= Area under the force-displacement curve.

Hence, for varying force the work done is equal to the definite integral of the force over the given displacement.

Work done in terms of rectangular Components

In the rectangular Components, the force and displacement vectors Can be Written as

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and $$\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$$

Therefore,

$$W = \vec{F}\cdot\vec{S}$$

$$= (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(S_x\hat{i} + S_y\hat{j} + S_z\hat{k})$$

$$\boxed{W = F_x S_x + F_y S_y + F_z S_z}$$

$$\left[\because \hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1,\ \hat{k}\cdot\hat{k} = 1 \text{ and } \hat{i}\cdot\hat{j} = 0,\ \hat{i}\cdot\hat{k} = 0,\ \hat{j}\cdot\hat{k} = 0\right]$$

Units of Work:—

1. Absolute units of work:— Work done is said to be one absolute unit if an absolute unit of force displaces a body through a unit distance in the direction of the force.

(a) Joule:— It is the absolute unit of work in SI, named after British Physicist " James Prescott Joule.

One joule of work is said to be done when a force of one newton displaces a body through a distance of one meter in its own direction.

i,e, 1 Joule = 1 newton x 1 metre

$$\therefore \boxed{1\,J = 1\,Nm}$$

(b) Erg:- It is the absolute unit of work in CGS System.

One erg of work is said to be done when a force of one dyne displaces a body through a distance of one Centimeter in its own direction.

i,e, 1 erg = 1 dyne x 1 cm
= 1 dyne cm.

Note:- $\boxed{1 \text{ joule} = 10^7 \text{ erg}}$

2. Gravitational unit of work :-

Work is said to be done one gravitational unit if a gravitational unit of force displaces a body through a unit distance in the direction of the force.

(a) Kilogram meter :-

It is the gravitational unit of work in SI.

"One kilogram meter of work is said to be done when a force of one kilogram weight displaces a body through one meter in its own direction."

i.e, 1 kg m = 1 kg wt × 1 m

= 9.80 N × 1 m

= 9.80 N m

= 9.80 J

(b) Gram centimeter :- It is gravitational unit of force in CGS system.

One gram centimeter of work is said to be done when a force of one gram weight displaces a body through one centimeter in its own direction.

i.e, 1 gm cm = 1 gm wt × 1 cm

= 980 dyne × 1 cm

= 980 dyne cm

= 980 erg.

Dimension of work :-

$$W = [M^1 L^1 T^{-2}][L^1]$$

$$= [M^1 L^2 T^{-2}]$$

This is the dimensional formula of work.

Nature of work :-

Work is a scalar quantity, because it is the dot product of force and displacement.

Energy:– Energy of a body is defined as its capacity or ability to do work.

The energy of a body is measured by the amount of work done the body can perform, therefore

(i) Like work, energy is a scalar quantity.

(ii) the dimensional formula of energy is same as that of work.

(iii) energy is measured in the same units as work.

Energy has several forms: Mechanical energy, heat energy, light energy, sound energy, chemical energy, solar energy etc.

Mechanical energy:–

Energy possessed by body by virtue of motion, change in configuration as well as interaction is called mechanical energy.

or The energy produced by mechanical means is called mechanical energy.

Mechanical energy has two forms–

(i) Kinetic energy

and (ii) Potential energy.

Kinetic Energy:–

The energy possessed by a body by virtue of its motion is called kinetic energy.

The amount of work that a moving body can do before coming to rest is equal to its kinetic energy.

Examples:-

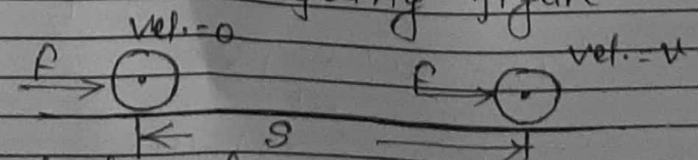
(i) A bullet fired from gun can pierce a target due to its kinetic energy.

(ii) The kinetic energy of air is used to run wind mills.

(iii) A fast moving stone can break a window pane. The stone has kinetic energy due to its motion and so it can do work.

(iv) The kinetic energy of a fast stream of water is used to run water mills. etc.

Expression for kinetic energy:– The kinetic energy of a body can be determined by calculating the amount of work required to bring the body into motion from its state of rest as shown in adjoining figure.



Consider a body of mass m initially at rest.

Let a constant force F applied on the body produces a displacement S in time t second, where its velocity is v, then

$$v^2 = u^2 + 2aS$$

$$or,\ v^2 = 0 + 2aS$$

$$or,\ a = v^2/2S$$

As the force and displacement are in same direction, so the work done is

$$W = FS$$
$$= ma \times S$$
$$= m\left(\frac{v^2}{2S}\right) \times S$$
$$= \frac{1}{2}mv^2$$

This work done appears as kinetic energy of the body

$$\therefore \boxed{K.E. = \frac{1}{2}mv^2}$$

Hence, the kinetic energy of a body is equal to one half the product of the mass of the body and the square of its velocity.

Alternate method :— Consider a body of mass m initially at rest. A constant force $\vec{F}$ applied on the body produces a displacement $\vec{ds}$ in its own direction in time dt where change in velocity is $d\vec{v}$. The small work done is

$$dW = \vec{F}\cdot\vec{ds}$$
$$= F\,ds\cos\theta$$
$$= F\,ds\cos 0^\circ = F\,ds$$

According to Newton's second law of motion

$$F = ma$$
$$= m\frac{dv}{dt}$$

$\left[\because a = \frac{dv}{dt} \text{ is the instantaneous acc}^n\right]$

$$So,\ dW = m\left(\frac{dv}{dt}\right)ds$$
$$= m\left(\frac{ds}{dt}\right)dv$$

Or, $dW = mv\,dv$ ——— (i)

$\left[\because \frac{ds}{dt} = v, \text{ instantaneous velocity}\right]$

The total work done to increase the velocity of a body from 0 to v is given by

$$W = \int dW = \int_0^v mv\,dv$$

$$= m\left[\frac{v^2}{2}\right]_0^v$$

$$= \frac{1}{2}m\left[v^2 - 0\right]$$

$$= \frac{1}{2}mv^2 \quad \text{——— (ii)}$$

This work done appears as the kinetic energy of the body

$$\therefore \boxed{K.E. = \frac{1}{2}mv^2}$$

Note:- The linear momentum of a body is

$$p = mv$$

Therefore,

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2m}\left(m^2v^2\right)$$

$$= \frac{1}{2m}\left(mv\right)^2$$

$$= \frac{1}{2m}p^2$$

$$\therefore \boxed{K.E. = \frac{p^2}{2m}}$$

This is the ~~kin~~ relation between kinetic energy and linear momentum of a body.

Work - Energy theorem

(i) For Constant force:— "It states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body."

Consider a constant force 'F' acting on a body of mass 'm' produces acceleration 'a' on it. After covering distance 's', suppose the velocity of the body changes from u to v. We use the equation of linear motion,

$$v^2 = u^2 + 2as$$

$$\text{or, } v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we get

$$\frac{1}{2}m(v^2 - u^2) = \frac{1}{2}m(2as)$$

$$\text{or, } \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

By Newton's second law, $ma = F$, the applied force.

Therefore

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$$

$$\text{or, } K_f - K_i = W$$

$$\therefore \boxed{W = K_f - K_i}$$

Hence, work done on the body by the net force is equal to change in kinetic energy of the body.

(ii) <u>For variable force:</u>—

Consider a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F} \cdot \vec{ds}$$

$$= F\,ds \cos 0°$$

$$= F\,ds$$

According to Newton's second law of motion

$$F = ma$$

$$= m\left(\frac{dv}{dt}\right)$$

$$\therefore dW = m\left(\frac{dv}{dt}\right)ds$$

$$= m\left(\frac{ds}{dt}\right)dv$$

$$= mv\,dv \quad \text{——— (1)}$$

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If the applied force increases the velocity from u to v, then total work done on the body will be

$$W = \int dW = \int_u^v mv\,dv$$

$$= m \int_u^v v\,dv$$

$$= m \left[\frac{v^2}{2}\right]_u^v$$

$$= \frac{1}{2} m \left[v^2 - u^2\right]$$

$$= \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

$$= K_f - K_i$$

$$\therefore \boxed{W = K_f - K_i = \text{Change in K.E. of the body.}}$$

Potential Energy:- Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or by its configuration.

Potential energy is also called mutual energy or energy of configuration.

Potential energy is measured by the amount of work that a body or system can do in passing from its present position or configuration to some standard position or configuration, called zero position or zero configuration.

Consider a body of mass 'm' lying on the surface of the earth. Let g be the acceln due to gravity at that place.

For height much smaller than the radius of the earth ($h \ll R$) the value g can be taken constant.

Force needed to lift the body up with zero acceleration

f = weight of the body

= mg

Work done on the body in raising it through height h,

W = mgh

This work done against gravity is stored as the potential energy (U) of the body.

$$\therefore \boxed{U = mgh}$$

At the surface of the earth, $h = 0$

$\therefore$ Gravitational P.E. at the earth's surface is zero.

i.e. $\boxed{U = 0}$